Transitioning Schedules for the Economic Lot Sizing Problem

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Abstract

A firm, which makes products with level demands on one work center, transitions, without unfilled orders, from their existing repeating schedule to a new one. If the inventory for all products is higher than that needed at any point in the new cycle then they need no transition period. The firm wants to minimize lost orders with a special transition period. The paper examines how to minimize this transition with no lost orders. A complete enumeration of all possible solutions is not always realistic. This paper shows a procedure to produce short and feasible transitions for the transition lot-sizing problem.

Key Words: *Scheduling, Order fulfillment.*

Introduction

Consider a firm that must make on one work center many products with constant demand rates, but can produce only one product at a time. In addition, the facility requires a setup time and/or cost to change its production from one product to another. The firm achieves this by building up inventories of products between their production runs. This facility wants to minimize its relevant costs, include inventory-holding, setup, and lost order costs. This problem, called the Economic Lot Scheduling Problem (ELSP), has been researched in many forms. Maxwell (1961) first formulated this classic ELSP problem with setup costs, as well as inventory costs and setup times. Later researchers extended the problem, such as Cheng, Yan, and Yang (1998) with variable production rates, Robinson and Sahin (2001) with overtime, Sriskandarajah, and Wagneur (1999) with lot streaming. Others came up with new solution techniques, such as genetic algorithms.

Whatever the solution technique, the general solution for the ESLP is to come up with a new repeating scheduling cycle, which consists of product lot sizes and a product sequence. Individual products may be made more than once in a cycle in different lot sizes. Thereafter we will mainly refer to production times as these are identical to lot sizes but are in the same units as setup times. Dobson (1987), for example, devised a heuristic that uses time-varying lot sizes for both the zero setup cost and the non-zero setup cost cases. The classic ELSP assumes that demand rate for each product is a constant for all time, and thus gives a steady state solution. Another extensive set of research considers the situation where future demand varies. Wolsey (1995, 1997) reviews some of the work done on this problem. Bradley and Conway (2003) give a tutorial on these problems.

Quite often, many firms use the steady state scheduling solutions when demand rates are not constant but are reasonably level. In such instances, infinite scheduling is often more desirable to the changing schedules that result from finite scheduling. Constant and repetitive production times and cycle sequences need far less administration. However, even with steady state demands, changes occur such as a new product introduction, an old product deletion, a process change, or changes in customer demands. So the firm calculates a new steady state schedule. Thus the facility requires a transition from the old ELSP solution to the new cycle schedule solution. This transition must be feasible, with minimum loss of orders.

The steady state scheduling research has failed to consider this transition problem. Steady state solution approaches implicitly assumed that starting the cycle and on-hand inventories are not part of the problem. In practice, there is this starting or transition problem. This situation is not dynamic, rather part of a punctuated equilibrium. Thus there is a transition to the static solution, followed by a period of static conditions, which in turn precedes another transition to a new steady state solution, *ad infinitum.*

Literature Survey

Need for a Transition

Anderson (1990) stated that most of the ELSP solutions imply that demand will always be met if the initial stocks are large enough. He reported that most researchers have supposed that the initial stocks of each product are indeed large enough to sustain the optimal production cycle. His question was whether the initial stocks are large enough to cover a certain level of demand. Anderson recommended letting setup and holding costs be zero. His problem then is determining the feasibility of a production schedule, given a current level of inventory, to meet a specific static demand for several products made on one machine.

However, Anderson's problem was not finding a feasible transition sequence to enable a regular repeating cycle to start, but rather a permanent repeating cycle that can start with no transition sequence. He concluded that the problem is NP-hard for both a finite and infinite horizon. Anderson then produced a heuristic procedure for finding a feasible schedule given initial inventory levels. He considered that a firm must determine the permanent repeating cycle from the initial inventory levels, not with a minimum cost function. Thus, Anderson's work does not use the work many researchers have done to find a near-optimal repeating cycle schedule.

Campbell and Mabert (1991) briefly mentioned the transition problem. They stated the solution is to adjust the lot sizes during the last period of the old cycle for a fixed horizon. They showed no examples, nor did they describe any detail about how to adjust the old cycle.

Breakdown Problem

Gallego (1988a, 1988b) considered a similar problem. He examined the case of an interruption, such as a machine breakdown, to a repeating cycle. His problem is how to adjust the planned schedule to make up the lost orders through backorders. Gallego called his problem the extended economic lot-scheduling problem (EELSP). His heuristic adjusts planned lot sizes after a small breakdown, after considering inventory, setup, and backorder costs. Gallego assumed that backordering was allowed, and he did not change the cycle sequence, but only the lot sizes. That is if there is a backorder of two and the normal lot size is ten then one should make twelve. If there is a stock of three then one should make seven.

Gallego argued that with many disruptions the optimal policy is a produce-up-to policy whenever the backorder costs are proportional to the processing times. However, his policy is to change lot sizes to make up for lost supply. This paper's transition heuristic tries to produce enough items during transition for the start of the new ELSP solution as we assume that backorders are not allowed.

We discuss first the literature then we give a full formulation of the general transition problem, including the full objective function. Next, we put forward our heuristic procedure, illustrated by a small example, carried out in detail to explain the procedure. Finally, we state our conclusions.

General Transition Problem Discussions

The Transition Problem

The general problem of transiting from one steady state with one regular ESLP sequence to another can be divided into several sub-problems. One is that the product mix does not change. In this paper we assume that only demand not product mix changes. (However, this formulation can easily handle the removal of a product(s) from the mix, and also may handle the addition of one product only). When demand changes there is probably a mismatch of inventories. This mismatch is what may or may not allow a smooth transition. There are several decision options during this transition.

1) Do you start transition now or wait to a better set of inventories occurs? (In the latter case there will be more possible solutions).
2) Can you choose where in the existing sequence to start the new schedule?
3) Can you change the new cycle sequence to suit the transition?
4) Can you use backorders to tide over the transitions?
5) Do you use a different transition sequence or must you go to the new regular sequence immediately.
6) If you cannot transition without lost orders do you minimize lost order cost, inventory cost, or lost order and inventory cost?
7) If you can transition without lost orders then do you minimize inventory cost or transition time?
8) Have the list of products changed or only the demand for each product.
9) If you are using an each product only once a cycle sequence, then do you change the sequence or only production times?

The results to these questions produce many different problems that we will briefly discuss below.

Simplest Transition

The simplest transition is when one keeps the cycle sequence the same as the existing sequence but only changes the production times (the old and new ESLP cycles use the same sequence as far as possible). This will work when one drops products but keeps the rest of the sequence constant). In this problem, the starting inventory, the required inventory at the end of the transition, the sequence, and the setup times are all known. All that is unknown are the transition production times. This problem is simple if there is a feasible answer.

First, if existing inventories for all products are greater than needed to start the new cycle then no transition is needed. The firm will have to use up surplus inventories by idling for the equivalent production time during the first few cycles of the new production cycle. Otherwise,

$$T == \Sigma_i \; (s_i + t_i) \quad \forall i \in \{1, \ldots, n\} \tag{1}$$

$$t_i = [\max(0, RI_i - EI_i) + d_i \, \Sigma_i \, [s_i * T] / p_i \tag{2}$$

Combining (1) and (2) gives,

$$T = \{\Sigma_i \, [s_i + \max(0, RI_i - EI_i) / p_i]\} / [1 - \Sigma_i \, (d_i / p_i)] \,\} \forall i \in \{1, \ldots , n\} \tag{3}$$

Where
- T = Total transition time
- i = product number
- n = number of products in new ESLP solution
- s_i = setup time for product i,
- d_i = demand rate for product i,
- p_i = production rate for product i,

t_i = transition production time for product i
RI_i = Required Inventory for product i at end of transition period
EI_i = Existing Inventory for product i

However, there are some complications. The first complication is if the existing inventory for product i is larger than ($RI_i + d_i$ □ T), then there is no need to produce product i. The effect is that you not only save production time but also a set up time (unless already set up for product i). Thus the transition times for the other products reduce, as they are a function of T. Therefore, you reduce the total transition time by more than the setup time for the not needed product.

The second complication is if a product will run out before its allotted time in the transition cycle. So one must check for all products that product i does not run out before it starts production again. That is that existing inventory will last its setup time and all the products' transition and setup times before product i restarts.

$$EI_i >= d_i * [\sigma_\iota + \Sigma_j^{i-1}(s_j + t_j)] \qquad (4)$$

where product j is produced before i.

One solution to this complication when product i runs out, would be to use short backorders within the transition period or accept the lost orders. One may also use a combination of backorders or lost orders to make a simple transition feasible. However, this paper considers the case where backorders or lost orders are not permitted, so if any product does not meet condition (4) then the simple transition schedule is unfeasible.

Complications

To solve this problem one could add any combination of three possible complications. The first obvious complication is to allow a change of sequence in the transition cycle. One would then examine n! transition sequences to see if any were feasible. If none were then one could choose the transition sequence that minimizes lost order or backorder costs.

The second complication that one could use to help the transition is changing the sequence of the new regular cycle, if the new cycle only makes each product once. If in this case there are n products, then one would have to examine n! possible starting sequences or sets of ending inventories.

The third complication is to choose when to start a transition rather than starting now. If one starts transition only when a setup is due then there are at least n sets of existing inventories to consider. If one uses all three complications then there are (n * n! * n!) possible solutions (n possible sets of existing inventories, n! possible transition sequences, and n! possible starting sequences or sets of ending inventories) to examine.

Complex Transition Problem

To sum up, the transition schedule, which consists of a product manufacturing sequence and a set of lot sizes for that sequence, has to meet the following constraints. First, the schedule has to start with the on-hand inventories when the old ELSP solution is abandoned. The first problem is when in the old ELSP cycle does the transition start? Sometimes, there is no choice because management wants to introduce the new schedule immediately. Second, the transition schedule must try to finish with at least enough of all products to enable the new ELSP solution to start with no stock-outs. Therefore, a variable in the problem is with what new product the new repeating cycle sequence should start. Third, the transition schedule should try to ensure that during the transition period all demand is met. Feasibility in this problem is defined as a transition that allows all demand to be met without backorders or stock-outs. With some transitions, there may be no feasible solution; then objective is to minimize the inevitable stock-outs.

Discussion of the objective

One could use the same objective as that for the classical ELSP; that is to minimize the cost rate of the transition. The transition cost rate is the total storage and setup costs, if any, of the transition period, divided by the transition's length. However, the main purpose of transition is to get to the new ELSP solution. A long transition with a cheap transition rate may not be preferable to a shorter transition with a higher rate if this cheap transition rate is higher than the cost rate for the new regular sequence. You also do not want a short cheap transition at the cost of a more costly ESLP sequence.

Thus the overall objective means that one should first generate the new ESLP first to minimize its cost, then work out the transition to get there. The second objective should be to minimize any lost order or backorder costs with a feasible transition. The third objective if there are choices of feasible transitions is to have a short as transition as is feasible. The fourth objective is to minimize the transition cost rate.

Other Factors

At the time the decision is made to change the cycle schedule, there has to be at most one product with zero inventory if there is to be a feasible transition. This should be the case as ELSP solutions only allow one product's inventory to run to zero at one time. However, as demand rates have changed there may be more than one zero inventory caused by increased demand or introduction of a new product. To start any new ESLP regular schedule without lost orders, there must be at least as much inventory as there would be at that point in the new regular schedule as if it had been running for some time.

Transition Problem

Examined Transition Problem

We now assume that a feasible transition solution is possible, that is without stock outs or backorders, to a new ELSP solution. Furthermore, the new repeating schedule ELSP solution can start at any point in its cycle. Thus the simplified objective is to minimize the feasible transition period to an already decided new ELSP solution. We assume that if a short feasible transition exists then it is unlikely that a longer feasible transition will have a lower cost rate, and if it does that that cost rate will be less than the new regular schedule.

Longer transition periods mean that more of the on-hand inventory is used, leading to longer production times and/or more setups in the transition, all of which should add to the cost. Managerially, one also wants to minimize the length of the transition period. Lopez and Kingsman (1991) stated that the demand is less likely to be constant over excessively long cycles. Regular cycles offer an advantage because they minimize management effort. A long transition period would negate this advantage, as the new regular cycle may then be used for only a short period before another transition is needed.

Regular Schedule Start Points

The first decision is where to start the new regular schedule. Many ELSP solution techniques describe only the regular schedule's production times not the regular cycle's sequence. If each product is made only once in one cycle and setup times and costs are not sequence dependent, then the regular sequence does not affect the cost rate. Therefore, with simple regular schedules, which only make each product once a cycle, the transition planning process must also decide the initial sequence of the regular schedule, as well as the transition's sequence and production times.

There is a multitude of possible points where one could start any repeating schedule, but this paper will consider, for simplification, to consider only those points in the regular cycle when production of a different product begins. Different regular cycle sequences will require different sets of required inventories of each product at their beginning to meet demand during the first regular cycle. A simple regular schedule ELSP solution for *n* products has *n* possible starting points. If the work center only makes each product once in each regular cycle, then there are (n!) possible initial regular cycle sequences. If the regular sequence is fixed, because the ELSP solution makes some products more than once a cycle or because of setup sequencing effects, then the number of sensible initial regular sequences is the same as the number of different lots in the regular cycle.

Number of Possible Transition Sequences

If the firm schedules each product at most only once in the transition sequence then there are (n!) possible transition sequences for each initial regular cycle sequence. This results in there being at least (n! ∗ n!) different combinations of initial regular and transition sequences. Not all these combinations are feasible. In some cases, there cannot be a feasible combination. If there are no feasible combinations, then the problem becomes one of minimizing the costs of stock outs as well as the time of the transition. One example is if a product that is not setup now has fewer inventories than its demand during its setup time. Another is if two products both have no inventory. This paper does not examine such cases further.

The number of combinations to examine soon becomes very large. For instance, finding the minimum transition time for four products involves comparing 576 combinations for a simple transition sequence. Transition sequences with repeating products would require considerably more comparisons. While it may be practicable to compare 576 sets, it is impracticable to compare the order of 10^{14} combinations that could exist for ten products. Consequently, it is necessary to have an efficient heuristic procedure to generate a more reasonable number of combinations.

Inventory Definition

We state all inventories in a common unit, which is days of demand. We define on-hand inventories as the amount of each product available when the decision is made to begin transition in days of new demand. We define the transition required beginning inventories for each product as the minimum amount of inventory that must be on-hand to allow the transition to occur without stock outs. The transition required beginning inventories are a function of the transition and regular schedule's sequences and production times. We define the required inventory for the regular cycle as the minimum amount of each product needed to start an initial sequence of the particular regular schedule without stock outs.

Transition Surpluses

If, for all products, the on-hand inventories are larger than the corresponding regular cycle beginning inventories, then there is no need for a transition period. A surplus for a product will occur when it is not made during transition, and its on-hand inventory minus its demand during the transition period is larger than the required beginning inventory for the regular schedule. One can use Gallego's produce-up-to policy in the initial regular cycles to burn off any surpluses.

Examined Problem Formulation

The transition sequence could have some products manufactured more than once in the sequence and some products not made at all. Each new production setup and time in the transition sequence is called a slot. When the firm must use a transition schedule then it must solve the following formulation for each combination of sequences. The combination problem formulation finds the best feasible lot sizes and is;

Minimize the transition period length,

$$\text{Min } T = s_{(y)} + \sum_{j=1}^{h} [s(j) + t(j)] \tag{5}$$

Where

j - number of a slot in the transition sequence
s(j) – setup time of product in jth slot
t(j) – transition production time of product in jth slot
x - name of product for which the workstation is now setup
y - name of the first product in the regular schedule initial sequence
h - the number of slots in the transition sequence
Note: Setup is not needed for first transition slot if product is already setup

$$s(1) = 0, \text{ if } x = pn(1) \tag{6}$$

Where pn(j) - is the name of the product in the jth slot of the transition sequence such that
For all products, there is enough inventory needed to start regular cycle

$$EI_i >= RI_i + d_i * T - p_i * t_i \tag{7}$$

$$\text{Where } t_i = \sum_{j=1}^{h} \{t(j) \text{ if } pn(j) = i, 0 \text{ o/w}\} \tag{8}$$

For all products, there must be enough existing inventory to last until production of that product starts in the transition if in the transition,

For all j = 1 to h, no stick outs before start of transition production
if product in slot j not made before in transition then,

$$EI_i >= d_{pn(j)} * \{s_{pn(j)} + \sum_{k=1}^{j-1} [s_{pn(k)} + t(k)]\} \tag{9}$$

Otherwise, if product i in jth slot was last made in the mth slot then,

$$t(m) * [p_i - d_i] >= d_i * \{s_i + \square \sum_{k=m+1}^{j-1} [s_{pn(k)} + t(k)]\} \tag{10}$$

Where k - counter in transition sequence

Solution Procedure

Branch and Bound

This paper suggests using a branch and bound procedure to determine both the new regular cycle sequence and the transition sequence, given a new ELSP solution. In this procedure, the new regular cycle sequences are the root nodes, which derive from existing inventories and regular cycle information. Transition sequences are the branch nodes, which come not only from the new regular cycle sequence, but also from the resultant inventory shortages. The first transition schedule is based on the inventory shortages that occur when there is no transition. Thereafter, the procedure branches from that short transition sequence by adding an additional product or products, which lengthens the total transition sequence time.

Branching stops in the following circumstances: when the program finds a feasible transition sequence; the current transition length exceeds the current bound; there is no feasible solution; or the transition has reached the managerially set maximum transition length. The current bound is the time of the shortest feasible transition sequence that the procedure has previously found.

Root Nodes

If there are *n* products, then there are *n!* possible new regular sequences for a simple ELSP solution. If the ELSP solution has a declared sequence of *m* production slots, then there are only *m* possible start nodes. This procedure considers each of these regular sequences as initial nodes. If the procedure can start with a regular cycle sequence that needs only a short transition time, then most of the root nodes thereafter will not branch. This would reduce the work involved considerably. However, it would be logical to start with a new regular sequence of most likely to run out first, that is smallest days of inventory left first.

Transition Initial Sequence

The main way to reduce the problem size is to assume that each product is made in the transition sequence at the most once. The firm could make products more than once in a transition, but this would increase the complexity of the procedure and most likely the length of the transition. For each regular cycle sequence and each set of transition products, there are different transition sequences worth considering. However, the most feasible sequence probably has the transition products placed in sequence of minimum days of equivalent demand for on-hand inventories plus setup time plus transition production time. In other words first make the product most likely to run out first. This is the first transition sequence that the procedure tries, called the transition initial lot sequence (TILS). Appendix A explains this in more detail.

Setup Time Savings

However, TILS ignores carry-over setup effects, which is that the transition saves a setup time when the already setup product is first in the transition sequence. So there are up to two transition sequences that the procedure considers at each branching. The first is TILS. The second is already setup product first, then TILS minus that product. Also putting the product made first in the regular sequence last in the transition makes no sense either.

Procedure Steps

The steps of the procedure are as follows:

1. Set the bound to the maximum length of permissible transition, normally L.
2. Convert the on-hand inventory into terms of days of equivalent demand.

$$DOBI_i = EI_i / d_i \text{ for all } i \tag{11}$$

Where $DOBI_i$ - the equivalent days of on-hand beginning inventory for product i

3. Find the accumulation ratio (a_i) of demand rates divided by production rates.

$$a_i = d_i / p_i \text{ for all } i \tag{12}$$

4. Find the new regular cycle product lot times, r_i, and total cycle length, L, by one of the many published methods (such as described by Bradley and Conway, 2003). A simple regular cycle consists of setup times and regular production times for each product assuming no slack time and no product repetition.

$$L = \Sigma_i (s_i + r_i) \text{ for all } i \tag{13}$$

The product produced in regular production time has to just equal the demand over the length of the regular cycle.

$$p_i * r_i = d_i * L \quad \text{for all } i \tag{14}$$

$$r_i = d_i * L/p_i = a_i * L \quad \text{for all } i \tag{15}$$

Combining (13) and (15) gives;

$$L = \Sigma_i (s_i) + L * \Sigma_i (a_i) = \Sigma_i (s_i) / [1 - \Sigma_i (a_i)] \tag{16}$$

5. If all on-hand inventories day equivalents are larger than the length of the regular cycle, then the problem is one of surpluses. Because there is no need for a transition, stop.

6. If two or more on-hand inventories are zero or a product other than the current setup item has less than its setup time in inventory, then there can be no feasible transition, so stop.

7. The first root node uses a new sequence of smallest $DOBI_i$ plus setup time plus regular cycle lot size (in days) first. This combines the two obvious rules; lowest inventory first (make it before it runs out), smallest lot size first (make the smallest lot time first as less likely to exhaust the other products). Then move those products with shortages forward to get other regular sequences to try.

8. Derive the shortages ($SHORT_i$) in days for each product i in this new regular sequence. A negative shortage is a surplus. The next equations show how to evaluate the shortages or surpluses for the any regular cycle sequence.

If product i is made first in the qth slot of the initial regular cycle, then for all i:

$$SHORT_i = s_i + \sum_{l=1}^{q-1} [s(l) + r(l)] - DOBI_i \tag{17}$$

Where q is the number of the slot in the new regular cycle sequence where product i is first made,

s(l) is the setup time of the product in the lth slot of the regular cycle,
r(l) is the regular production time of the product in the lth slot.

Note: $s(1) = 0$, if product in slot 1 is already setup (18)

9. The problem is one of surpluses if all the $SHORT_i$s' values are negative or zero for this sequence. Then there is no need for a transition and the procedure stops. Such surpluses can be “burnt up” using idle time in the new regular sequence.

10. Otherwise, for each transition set, find the transition lot sizes (t_i) and transition time period (T) using:

$$T = \Sigma_i (s_i + t_i), \text{ for all } i \text{ whose } SHORT_i > 0 \tag{19}$$

Where $S_x = 0$, if $pn(1) = x$

$$t_i = (SHORT_i + T) * a_i \tag{20}$$

Which combine to form

$$T = [\Sigma_i (s_i + SHORT_i * a_i)] / [1 - \Sigma_i (a_i)] \tag{21}$$

11. Thus, find the TILS, smallest ($t_i + s_i + DOBI_i$) first. Try TILS first, then try, if the already setup product is in transition, that setup product first then TILS sequence. Recalculate the lot sizes and transition length each time. Note do not use a transition sequence that has product that is made first in the regular sequence last in the transition sequence.

12. If the transition length is more than the bound, then go back to next root regular sequence in step seven.

13. Check the external feasibility of this transition sequence by ensuring that:

$$SHORT_i >= T, \text{ for all i whose } SHORT_i >= 0 \quad (22)$$

If some products not in the transition run out before the transition ends, then try the next sequence at step ten.

14. Check the internal feasibility of this transition sequence.

$$DOBI_i(j) >= s_i(j) + \sum_{k=1}^{i-1} [s_i(k) + t_i(k)], \text{ for all k where } m_j = 0 \quad (23)$$

$$\text{Where } s_x = 0, \text{ if } pn(j) = x \quad (24)$$

If some products in the transition run out before they can start production in the transition, then try the next sequence at step ten.

15. Record the transition length if the transition is feasible. If this is less than the bound, then record the length as the new bound and record transition and regular cycle details of the best transition so far found. Go to the next regular sequence root node at step seven.

16. If all sequences for a transition fail their external feasibility, then add the new shortage ($SHORT_i < L$) products to the transition set and branch again at step nine.

17. If all sequences for a transition have a transition length greater than the bound or fail their internal feasibility test, then stop and try the next regular sequence at step seven.

18. If no feasible set of sequences produces a transition time less than the initial bound after trying all the root nodes, then stop and report that there are no feasible solutions.

Advantages of this Procedure

This procedure needs fewer transition length derivations than full enumeration does. For n products there are *n!* possible initial nodes. In this procedure, each of these initial nodes has up to four possible initial sub-branches. Thereafter, each offspring node has at most ($2n^2$) possible sub-branches. However, most likely there are far fewer sub-branches at each node. Thus the maximum number of nodes is ($n! * 2 * n^2$). If one has ten products then one has a maximum of 725,760,000 (10! ∗ 200) nodes. This number is considerably less than the 10^{14} nodes of the complete enumeration method. In most cases, there will be great deal less sensible nodes than this.

Example Results

To demonstrate the use of this procedure, consider the following small example of four products made in one work center. Table 1 has the required input data for each of the four products.

Table 1 - First Example with Four Products - Input Details						
PRODUCT INPUT DETAILS						
Product Name i	Daily Sales Rate d_i	Daily Production Rate p_i	Days of Existing Inventory $DOBI_i$	Setup in Days s_i	Accumulation Ratio a_i	Regular Production Time (days) r_i
1	1250	2500	12.850	1	.500	15.385
2	583	2500	19.803	1	.2332	7.175
3	267	2500	2.951	1	.1068	3.286
4	75	2500	15.040	1	.0300	.923
Product 1 is already setup.						

The procedure carries on as follows:

1. A set of regular cycle time lot times is calculated based on the minimum cycle that is feasible with each product only made once a cycle. This cycle has a cycle length of 30.769 days. Table 1 shows the regular cycle production times for each product.

2. All products have on-hand inventory less than any new regular cycle length. None are zero. Therefore, this example may need a transition. The bound is 31 days.

3. The procedure tries first an initial root node of regular cycle sequence 3421, which is the reverse order of the product on-hand inventories, plus setups, plus regular cycle lot times. All products are in deficit, with $\Sigma_i (a_i) = .87$. Table 2 shows the details.

Table 2. - Initial Feasible Results for First Example with Four Products						
INITIAL REGULAR CYCLE						
Position Number	Product Number	Setup Time	Lot Time	Starting Inventory	Needed Inventory	Shortage
1	3	1	3.286	2.951	1	-1.951
2	4	1	.923	15.040	5.286	-9.754
3	2	1	7.175	19.803	7.209	-12.594
4	1	1	15.385	12.850	15.384	**2.534**
The regular schedule cycle time is 30.769 days.						
INITIAL TRANSITION CYCLE						
1	3	1	5.161	2.951	1	-1.951
2	1	1	.623	12.850	7.161	-5.689
The initial transition period length is					**7.784**	
All units are in days.						

4. Table 2 shows that Product One is in deficit. Therefore, the procedure branches to a transition set of only Product One. This gives a transition time of 2.53 days.

5. However, Product Three has a DOBI of 1.92 days, which is less than the transition time of 2.53. Thus, the procedure branches to a transition set with Product One and Product Three. The procedure tries Product Three first because it has a smaller product on-hand inventory, plus setup time, plus regular cycle lot time total than Product One. Incidentally, if Product One is first, then Product Three will be both the last in the transition and first in the regular sequence, which is really back to a transition of Product One only. This initial feasible solution takes 7.784 days, which is the new bound. Table 2 contains the details.

6. The procedure next tries as a root node the initial regular cycle 3412, which reverses the sequence of the last two regular products. This advances the previous short product. Table 3 shows the details.

Table 3. - Best Feasible Results for First Example with Four Products

Position Number	Product Number	Setup Time	Lot Time	Starting Inventory	Needed Inventory	Shortages
INITIAL REGULAR CYCLE						
1	3	1	3.286	2.951	1	-1.951
2	4	1	.923	15.040	5.286	-9.754
3	1	1	15.385	12.850	12.850	-5.641
4	2	1	7.175	19.803	19.803	**3.791**
The regular schedule cycle time is 30.769 days.						
INITIAL TRANSITION CYCLE						
1	3	1	.224	2.951	1	-1.951
2	2	1	1.830	19.803	2.224	-17.579
The initial transition period length is						**4.054**
All units are in days						

4. Table 3 shows that Product Two is in deficit. Therefore, the procedure branches to a transition set of only Product Two. This gives a transition time of 2.45 days.

5. However, Product Three has a DOBI of 1.92 days, which is less than the transition time of 2.53. Thus, the procedure branches to a transition set with Product Two and Product Three. The procedure tries Product Three first because it has a smaller product on-hand inventory, plus setup time, plus regular cycle lot time total than Product Two. Incidentally, if Product Two is first, then that sequence will also be feasible. This feasible solution takes 4.05 days, which is the new bound. Table 3 contains the details. We then used this procedure on a larger example with ten products. The procedure found the best combination of sequences on the fourth iteration.

Conclusions

Firms can generate steady state production scheduling solutions for level demands of many products made on one work center. However, phasing in such a schedule from a specific on-hand inventory set may require a transition period. There has been little research on this. A transition period alternating with steady state periods may be more representative of reality than the classic dynamic problems, which typically have implied zero demand ending assumptions. This paper examined how a firm plans the transition from a given set of on-hand inventories to a new regular cycle. This problem specifically is to determine the transition sequence and production times and the initial regular cycle sequence, without any lost orders if possible.

One approach to finding the best solution would be a rigorous examination of all the possible combinations of transition sequences and initial regular cycle sequences. This paper has shown that often this is too large a task for a reasonably sized problem. Consequently, there is a need for a procedure that will give a good solution. The procedure with the demonstration example gave a feasible answer with a short transition period with little effort. Managers have to plan how they would changeover between two repeating production cycle schedules. Firms need a procedure to do this. This paper's procedure assumed that the changeover had to start at a given time. In practice, managers may have the choice of when to change from one regular cycle to another. In that case, there would be a choice of sets of on-hand inventories at the transition point. This would complicate the decision because it would multiply by n the number of possible solutions to the problem.

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Appendix A

This appendix is to demonstrate that if one ignores all effects of saving on setups, then the TILS (Transition Initial Lot Sequence) sequence gives the shortest transition length, if one ignores the saved setup time effect. The TILS is a sequence that puts first the product with the minimum transition production time in days, plus the setup time in days, plus the beginning inventory in days of equivalent demand. Furthermore, it demonstrates that if the TILS sequence is infeasible, then one cannot produce a feasible sequence by exchanging product positions.

Consider two products in the transition sequence, named A and B. The transition sequence is in TILS, A then B. Let X be the transition length up to the finish of production of the product before A. Then the following equations are true:

$$DOBI_a + s_a + t_a < DOBI_b + s_b + t_b \quad (A.1)$$

Assume that with this sequence, product A does not run out but Product B does, that is the TILS is infeasible. Then the following are true:

$$X <= DOBI_a \quad \text{(A.2)}$$

$$X + s_a + t_a > DOBI_b \quad \text{(A.3)}$$

Rearranging equation (A.3) gives:

$$s_a + t_a + DOBI_a > DOBI_a + DOBI_b - X \quad \text{(A.4)}$$

Combining equation (A.4) with equation (A.1) gives:

$$DOBI_b + s_b + t_b > s_a + t_a + DOBI_a > DOBI_a + DOBI_b - X \quad \text{(A.5)}$$

$$DOBI_b + s_b + t_b > DOBI_a + DOBI_b - X \quad \text{(A.6)}$$

Simplifying equation (A.6) gives:

$$s_b + t_b > DOBI_a - X \quad \text{(A.7)}$$

Rearranging equation (A.7) gives:

$$X + s_b + t_b > DOBI_a \quad \text{(A.8)}$$

The question now is can we improve on this infeasible TILS? Assume that swapping the positions of products A and B would make this sequence feasible. Then the following equations must be true:

$$X <= DOBI_b \quad \text{(A.9)}$$

$$X + s_b + t_b <= DOBI_a \quad \text{(A.10)}$$

Equation (A.10) is not possible because it is incompatible with equation (A.9). Thus, we cannot make this sequence feasible by swapping these products. Therefore, TILS is the sequence most likely to be feasible.